Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE Mathematics
Core Mathematics 3 (6665)

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\text{ where}|pq|=|c|$, leading to x = …

$(ax^2+bx+c)=(mx+p)(nx+q),\text{ where}|pq|=|c|\text{ and }|mn|=|a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n\rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n\rightarrow x^{n+1}$)

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	$x^2-9=(x+3)(x-3)$	B1
	$\dfrac{4x}{x^2-9}-\dfrac{2}{(x+3)}=\dfrac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$=\dfrac{2x+6}{(x+3)(x-3)}$	A1
	$=\dfrac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$=\dfrac{2}{(x-3)}$	A1
		(4)

B1 $x^2-9=(x+3)(x-3)$ This can occur anywhere.

M1 For combining the two fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

For example accept $\dfrac{4x}{x^2-9}-\dfrac{2}{x+3}=\dfrac{4x(x+3)-2(x^2-9)}{(x+3)(x^2-9)}$

accept separately $\dfrac{4x}{(x+3)(x-3)}-\dfrac{2}{(x+3)}=\dfrac{4x}{(x+3)(x-3)}-\dfrac{2x-3}{(x+3)(x-3)}$ condoning missing bracket

condone $\dfrac{4x}{x^2-9}-\dfrac{2}{x+3}=\dfrac{4x(x+3)-2}{(x+3)(x^2-9)}$ ....as only one numerator has been adapted

A1 A correct intermediate form of $\dfrac{\text{simplified linear}}{\text{simplified quadratic}}$

Accept $\dfrac{2x+6}{(x+3)(x-3)}$, $\dfrac{2x+6}{x^2-9}$, and even $\dfrac{(2x+6)\cancel{(x+3)}}{(x^2-9)\cancel{(x+3)}}$,

A1 Further factorises and cancels (which may be implied) to reach the answer $\dfrac{2}{x-3}$

Do not penalise correct solutions that include incomplete lines Eg $\dfrac{4x-2(x-3)}{(x+3)(x-3)}=\dfrac{4x-2x+6}{\dots}=\dfrac{2x+6}{(x+3)(x-3)}=\dfrac{2}{x-3}$

This is not a "show that" question.

Note: Watch out for an answer of $\dfrac{2}{x+3}$ probably scored from $\dfrac{4x-2(x-3)}{(x+3)(x-3)}=\dfrac{2x-6}{(x+3)(x-3)}=\dfrac{2(x-3)}{(x+3)(x-3)}$

This would score B1 M1 A0 A0

Question Number	Scheme	Marks
2.(a)	$e^{3x-9}=8 \Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\dfrac{\ln 8+9}{3}, =\ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\dfrac{2y+5}{4-y}\right)=2$	M1
	$\left(\dfrac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\dfrac{4e^2-5}{2+e^2}$	dM1, A1
		(4)
		7 marks

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8+9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

....

Alt I (a)

$e^{3x-9}=8\Rightarrow \dfrac{e^{3x}}{e^9}=8\Rightarrow e^{3x}=8e^9\Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8}\Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2\Rightarrow x-9=\ln 2$)

....

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the e^2

M1 Scored for an attempt to undo the ln's to get an equation in y This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2\Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making y the subject. Expect to see both terms in y collected and factorised (may be implied) before reaching y =. Condone slips, for eg, on signs. y =2.615 scores this.

A1 $y=\dfrac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\dfrac{13}{2+e^2}$ ISW after you see the correct answer.

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Special Case: $\ln(2y+5)-\ln(4-y)=2\Rightarrow\dfrac{\ln(2y+5)}{\ln(4-y)}=2\Rightarrow\dfrac{2y+5}{4-y}=e^2\Rightarrow$ Correct answer score M0 M1 M1 A0

Question Number	Scheme	Marks
3.(a)	$y \geqslant 3$	B1
		(1)
(b)	$y = 3+\sqrt{x+2} \Rightarrow y-3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x \geqslant 3$	A1
		(3)
(c)	$g(x) = x \Rightarrow 3+\sqrt{x+2} = x$	
	$\Rightarrow x+2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \dfrac{7 \pm \sqrt{21}}{2} \Rightarrow x = \dfrac{7+\sqrt{21}}{2}$ only	M1, A1
		(4)
(d)	$a = \dfrac{7+\sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(c) Alt	Solves $g^{-1}(x) = x \Rightarrow (x-3)^2 - 2 = x$	
	$\Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \dfrac{7 \pm \sqrt{21}}{2} \Rightarrow x = \dfrac{7+\sqrt{21}}{2}$ only	dM1, A1
		(4)

(a)

B1 States the correct range for g Accept $g(x) \geqslant 3, g \geqslant 3$, Range $\geqslant 3, [3, \infty)$ Range is greater than or equal to 3

Condone $f \geqslant 3$ Do not accept $g(x) > 3, x \geqslant 3, (3, \infty)$

(b)

M1 Attempts to make *x* or a swapped *y* the subject of the formula. The minimum expectation is that the 3 is moved over followed by an attempt to square both sides. Condone for this mark $\sqrt{x+2} = y \pm 3 \Rightarrow x+2 = y^2 \pm 9$

A1 Achieves $x = (y-3)^2 - 2$ or if swapped $y = (x-3)^2 - 2$ or equivalent such as $x = y^2 - 6y + 7$

A1 Requires a correct function in *x* + correct domain **or** a correct function in *x* with a correct follow through on the range in (a) but do not follow through on $x \in \mathbb{R}$

Accept for example $\mathrm{g}^{-1}(x) = (x-3)^2 - 2, \quad x \geqslant 3$ Condone $\mathrm{f}^{-1}(x) = (x-3)^2 - 2, \quad x \geqslant 3$

or variations such as $y = (x-3)^2 - 2, \quad x > 3$ if (a) was $y > 3$

Accept expanded versions such as $\mathrm{g}^{-1}(x) = x^2 - 6x + 7, \quad x \geqslant 3$ but remember to isw after a correct answer (Condone $\mathrm{f}^{-1}(x) = x^2 - 6x + 7, \quad x \geqslant 3$)

(c)

M1 Sets $3 + \sqrt{x+2} = x$, moves the 3 over and then attempts to square both sides.

Can be scored for $\sqrt{x+2} = x - 3 \Rightarrow x + 2 = x^2 \pm 9$

A1 $x^2 - 7x + 7 = 0$. The = 0 may be implied by subsequent working

M1 Correct method of solving their 3TQ by the formula/ completing the square. The equation must have real roots.

It is dependent upon them having attempted to set $3 + \sqrt{x+2} = x$ and proceeding to a quadratic.

You may just see both roots written down which is fine.

Allow for this mark decimal answers Eg 5.79 and 1.21 for $x^2 - 7x + 7 = 0$ You may need to check with a calc.

A1 $(x) = \dfrac{7+\sqrt{21}}{2}$ or exact equivalent **only**.

This answer following the correct quadratic would imply the previous M

Allow $x = \dfrac{7}{2} + \sqrt{\dfrac{21}{4}}$ but **DO NOT** allow $x = \dfrac{7 \pm \sqrt{21}}{2}$

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(c) can of course be attempted by solving

$$3 + \sqrt{x+2} = \text{"}(x-3)^2 - 2\text{"} \Rightarrow x^4 - 12x^3 + 44x^2 - 49x + 14 = 0$$
$$\vdots \qquad \Rightarrow (x^2 - 7x + 7)(x^2 - 5x + 2) = 0$$

The scheme can be applied to this

....

(d)

B1ft $(a) = \dfrac{7+\sqrt{21}}{2}$ oe . You may condone $\boldsymbol{x} = \dfrac{7+\sqrt{21}}{2}$. You may allow this following a re - start.

You may allow the correct decimal answer, awrt 5.79, following exact/decimal work in part (c) or a restart.

Follow through on their root, including decimals, coming from the **positive** root with the **positive** sign in (c).

Eg In (c) . $x^2 - 7x + 11 = 0 \Rightarrow x = \dfrac{7 \pm \sqrt{5}}{2}$ So the correct follow through would be $x = \dfrac{7+\sqrt{5}}{2}$

If they only had one root in (c) then follow through on this as long as it is positive.

SC. If they give the correct roots in parts (c) and (d) without considering the correct answer then award B1 in (d) following the A0 in (c). So $(x) = \dfrac{7 \pm \sqrt{21}}{2}$ as their answer in part (c), allow $(x/a) = \dfrac{7 \pm \sqrt{21}}{2}$ for B1 in (d).

Question Number	Scheme	Marks
4.(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\dfrac{2}{5}\Rightarrow\alpha=\text{awrt } 0.381$	M1A1
		(3)
(b)	$5\cot 2x-3\operatorname{cosec} 2x=2\Rightarrow 5\dfrac{\cos 2x}{\sin 2x}-\dfrac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\dfrac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\dfrac{3}{\sqrt{29}}\right)\Rightarrow x=\ldots$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)
		(9 marks)
Alt I (c)	$5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$	
	$\Rightarrow 4\tan^2 x+2\tan x-1=0$	M1
	$\Rightarrow\tan x=\dfrac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)
Alt II (c)	$5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ **&** $\cos^2 2x=1-\sin^2 2x$	
	$\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$	M1
	$\Rightarrow\sin 2x=\dfrac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1 $\tan\alpha=\pm\dfrac{2}{5},\ \tan\alpha=\pm\dfrac{5}{2}\Rightarrow\alpha=\ldots$

If R is used to find α accept $\sin\alpha=\pm\dfrac{2}{R}$ or $\cos\alpha=\pm\dfrac{5}{R}\Rightarrow\alpha=\ldots$

A1 $\alpha =$ awrt 0.381 Note that the degree equivalent $\alpha =$ awrt $21.8°$ is A0

(b)

M1 Replaces $\cot 2x$ by $\dfrac{\cos 2x}{\sin 2x}$ **and** $\operatorname{cosec} 2x$ by $\dfrac{1}{\sin 2x}$ in the lhs

Do not be concerned by the coefficients 5 and -3.

Replacing $\cot 2x$ by $\dfrac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\dfrac{\sin 2x}{\cos 2x}$

They may state $\times \sin 2x \Rightarrow 5\cos 2x - 3 = 2\sin 2x$ which implies this mark

A1 cso $5\cos 2x - 2\sin 2x = 3$ There is no need to state the value of 'c'

The notation must be correct. They cannot mix variables within their equation

Do not accept for the final A1 $\tan 2x = \dfrac{\sin}{\cos} 2x$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their R and α from part (a) and their c from part (b)

Accept $\cos(2x \pm {'\alpha'}) = \dfrac{'c'}{'R'}$ Condone $\cos(\theta \pm {'\alpha'}) = \dfrac{'c'}{'R'}$ or even $\cos(x \pm {'\alpha'}) = \dfrac{'c'}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$

Don't be concerned if they change the variable in the question and solve for $\theta =$ (as long as all operations have been undone).You may not see any working. It is implied by one correct answer.

You may need to check with a calculator.

Eg for an incorrect α $\cos(2x + 1.19) = \dfrac{3}{\sqrt{29}} \Rightarrow x = -0.105$ would score M1 dM1 A0 A0

A1 One solution correct, usually $x = 0.3/0.30$ or $x = 2.46$ or in degrees $17.2°$ **or** $141.(0)°$

A1 Both solutions correct awrt $x =$ awrt $0.30, 2.46$ and no extra values in the range.

Condone candidates who write 0.3 and 2.46 without any (more accurate) answers

In degrees accept awrt 1 dp $17.2°, 141.(0)°$ and no extra values in the range.

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Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.

They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their R and α from part (a)

Accept $\cos(2x \pm {'\alpha'}) = \dfrac{2}{'R'}$ Condone $\cos(\theta \pm {'\alpha'}) = \dfrac{2}{'R'}$ or even $\cos(x \pm {'\alpha'}) = \dfrac{2}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$

You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an correct α and R $\cos(2x + 0.381) = \dfrac{2}{\sqrt{29}} \Rightarrow x = 0.405$

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Alt to part (c)

M1 Attempts both double angle formulae condoning sign slips on cos2x , divides by $\cos^2 x$ and forms a quadratic in tan by using the identity $\pm 1 \pm \tan^2 x = \sec^2 x$

dM1 Attempts to solve their quadratic in tan$x$ leading to a solution for x.

A1 A1 As above

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Question Number	Scheme	Marks
5. (a)	At P $x=-2 \Rightarrow y=3$	B1
	$\dfrac{dy}{dx}=\dfrac{4}{2x+5}-\dfrac{3}{2}$	M1, A1
	$\left.\dfrac{dy}{dx}\right\vert_{x=-2}=\dfrac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\dfrac{2}{5}\left(x-(-2)\right)$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\dfrac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\dfrac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\dfrac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
(c)	Substitutes $x_1=2 \Rightarrow x_2=\dfrac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929$.	A1
		(2)
		(10 marks)

(a)

B1 $y=3$ at point *P*. This may be seen embedded within their equation which may be a tangent

M1 Differentiates $\ln(2x+5) \to \frac{A}{2x+5}$ or equivalent. You may see $\ln(2x+5)^2 \to \frac{A(2x+5)}{(2x+5)^2}$

A1 $\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$ oe. It need not be simplified.

M1 For using a correct method of finding the equation of the normal using their numerical value of $-\left.\frac{dx}{dy}\right|_{x=-2}$ as the gradient. Allow for $(y-'3')=-\left.\frac{dx}{dy}\right|_{x=-2}(x--2)$, oe.

At least one bracket must be correct for their $(-2,3)$

If the form $y=mx+c$ is used it is scored for proceeding as far as *c* = ..

A1 $\pm k(5y+2x=11)$ It must be in the form $ax+by=c$ as stated in the question

Score this mark once it is seen. Do not withhold it if they proceed to another form, $y=mx+c$ for example

If a candidate uses a graphical calculator to find the gradient they can score a maximum of B1 M0 A0 M1 A1

(b)

M1 For combining 'their' **linear** $5y+2x=11$ with $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in just *x*, condoning slips on the rearrangement of their $5y+2x=11$. Eg $2\ln(2x+5)-\frac{3x}{2}=\frac{11\pm 2x}{5}$ is OK

dM1 Collects the two terms in *x* and proceeds to $ax=b\ln(2x+5)+c$ Allow numerical slips

A1* This is a given answer. All aspects must be correct including bracketing

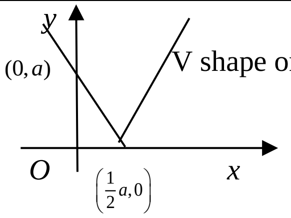
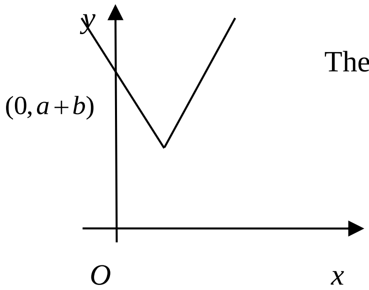
(c)

M1 Score for substituting $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln(2\times 2+5)-2$ or exact equivalent

This may implied by $x_2=1.99...$

A1 Both values correct. Allow awrt $x_2=1.9950$ and $x_3=1.9929$ but condone $x_2=1.995$

Ignore subscripts. Mark on the first and second values given.

Question Number	Scheme	Marks
6.(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ **and** $(0,a)$	B1
	Correct shape, position and coordinates	B1
(ii)	 Their "V" shape translated up or $(0,a+b)$	B1ft
	Correct shape, position and $(0,a+b)$	B1
		(4)
(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\dfrac{3}{2}x+8$ in either x or with $x=$ c	
	$2c-a+b=\dfrac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$	M1
	Combines $kc=\mathrm{f}(a,b)$ with $a+b=8$ $\Rightarrow c=4a$	dM1 A1
		(4)
		(8 marks)

(a)(i)

B1 V shape sitting anywhere on the x- axis **or** for $\left(\frac{1}{2}a,0\right)$ and $(0,a)$ lying on the curve.

Condone non -symmetrical graphs and ones lying on just one side of the y -axis

B1 V shape sitting on the positive x-axis at $\left(\frac{1}{2}a,0\right)$, cutting the y-axis at $(0,a)$ and lying in both quadrants 1 and 2

Accept $\dfrac{1}{2}a$ and a marked on the correct axis. Condone say $(a,0)$ for $(0,a)$ as long as it is on the correct axis.

Condone a dotted line appearing on the diagram as many reflect $y=2x-a$ to sketch $y=\lvert 2x-a\rvert$

If it is a solid line then it would not score the shape mark.

(a)(ii)

B1ft Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.

Alternatively score for the $(0,a+b)$ lying on the curve

B1 V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the y- axis at $(0,a+b)$

Ignore any coordinates given for the vertex.

(b)

B1 States or uses $a+b=8$ or exact equivalent. Condone use of capital letters throughout

It is not scored for just $|0-a|+b=8$

M1 This M is for an understanding of the modulus.

It is scored for an attempt at solving $(2x-a)+b=\frac{3}{2}x+8$ or $-(2x-a)+b=\frac{3}{2}x+8$ in either x or with x replaced by c. The signs of the $2x$ and the a must be different. $|2x-a| \neq 2x+a$

You may see $(2x-a)+b=\frac{3}{2}x+8 \Rightarrow kx=\mathrm{f}(a,b)$

You may see $-2x+a+b=\frac{3}{2}x+8 \Rightarrow kx=\mathrm{f}(a,b)$

You may see $(2x-a)+b=\frac{3}{2}x+8 \Rightarrow kx=\mathrm{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

You may see $-2c+a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

dM1 This dM mark is scored for combining $b=8-a$ with $(2x-a)+b=\frac{3}{2}x+8$ (or their $kx=\mathrm{f}(a,b)$ resulting from that equation) resulting in a link between x and a **Both equations must have been correct initially.**

Alternatively for combining $b=8-a$ with their $2c-a+b=\frac{3}{2}c+8$ (or their $kc=\mathrm{f}(a,b)$ resulting from that equation) resulting in a link between c and a

You may condone sign slips in finding the link between x (or c) and a

If you see an approach that involves making $|2x-a|$ the subject followed by squaring, and you feel that it deserves credit, please send to review. The solution proceeds as follows

Look for $|2x-a|=\frac{3}{2}x+8-b \Rightarrow |2x-a|=\frac{3}{2}x+a \Rightarrow (2x-a)^2=\left(\frac{3}{2}x+a\right)^2 \Rightarrow 7x\left(\frac{1}{4}x-a\right)=0$

A1 $c=4a$ ONLY

....

Special Case where they have the roots linked with the incorrect branch of the curve.

They have $x=0$ as the solution to $2x-a+b=\frac{3}{2}x+8 \Rightarrow -a+b=8$..................(1)

They have $x=c$ as the solution to $-2x+a+b=\frac{3}{2}x+8 \Rightarrow \frac{7}{2}x=a+b-8$............(2)

Solve (1) and (2) $\Rightarrow x=\frac{4}{7}a$

Hence $\Rightarrow c=\frac{4}{7}a$

This would score B0 M1 dM0 A0 anyway but should be awarded SC B0, M1 dM1, A0 for above work leading to either $x=\frac{4}{7}a$ or $c=\frac{4}{7}a$

Question Number	Scheme	Marks
7(i) (a)	$y=2x\left(x^2-1\right)^5 \Rightarrow \dfrac{dy}{dx}=\left(x^2-1\right)^5\times 2+2x\times 10x\left(x^2-1\right)^4$ $\Rightarrow \dfrac{dy}{dx}=\left(x^2-1\right)^4\left(2x^2-2+20x^2\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$	M1A1 M1 A1 (4)
(b)	$\dfrac{dy}{dx}\ldots 0 \Rightarrow \left(22x^2-2\right)\ldots 0 \Rightarrow$ critical values of $\pm\dfrac{1}{\sqrt{11}}$ $x\ldots\dfrac{1}{\sqrt{11}}\quad x,, -\dfrac{1}{\sqrt{11}}$	M1 A1 (2)
(ii)	$x=\ln(\sec 2y)\Rightarrow \dfrac{dx}{dy}=\dfrac{1}{\sec 2y}\times 2\sec 2y\tan 2y$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2\tan 2y}=\dfrac{1}{2\sqrt{\sec^2 2y-1}}=\dfrac{1}{2\sqrt{e^{2x}-1}}$	B1 M1 M1 A1 (4) **10 marks**
Alt 1 (ii)	$x=\ln(\sec 2y)\Rightarrow \sec 2y=e^x$ $\Rightarrow 2\sec 2y\tan 2y\dfrac{dy}{dx}=e^x$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{e^x}{2\sec 2y\tan 2y}=\dfrac{e^x}{2e^x\sqrt{\sec^2 2y-1}}=\dfrac{1}{2\sqrt{e^{2x}-1}}$	 B1 M1M1A1 **(4)**
Alt 2 (ii)	$y=\dfrac{1}{2}\arccos\left(e^{-x}\right)\Rightarrow \dfrac{dy}{dx}=-\dfrac{1}{2}\times\dfrac{1}{\sqrt{1-\left(e^{-x}\right)^2}}\times -e^{-x}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2\sqrt{e^{2x}-1}}$	B1M1M1 A1 **(4)**

(i)(a)

M1 Attempts the product rule to differentiate $2x(x^2-1)^5$ to a form $A\left(x^2-1\right)^5+Bx^n\left(x^2-1\right)^4$ where $n=1$ or 2. and A, B >0 If the rule is stated it must be correct, and not with a "$-$" sign.

A1 Any unsimplified but correct form $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=2\left(x^2-1\right)^5+20x^2\left(x^2-1\right)^4$

M1 For taking a common factor of $\left(x^2-1\right)^4$ out of a suitable expression

Look for $A\left(x^2-1\right)^5 \pm Bx^n\left(x^2-1\right)^4=\left(x^2-1\right)^4\left\{A\left(x^2-1\right)\pm Bx^n\right\}$ but you may condone missing brackets

It can be scored from a *vu'*-*uv'* or similar.

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$ Expect $\mathrm{g}(x)$ to be simplified but accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4 2\left(11x^2-1\right)$

There is no need to state g(x) and remember to isw after a correct answer. This must be in part (a).

(i)(b)

M1 Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}\geqslant 0, >0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and proceeds to find one of the critical values for **their** $\mathrm{g}(x)$ or their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ rearranged and $\div\left(x^2-1\right)^4$ if g(x) not found. $\mathrm{g}(x)$ should be at least a 2TQ with real roots. If g(x) is factorised, the usual rules apply. The M cannot be awarded from work **just** on $\left(x^2-1\right)^4\geqslant 0$ ie $x=\pm1$

You may see and accept decimals for the M.

A1 cao $x\geqslant\frac{1}{\sqrt{11}}$ $x\leqslant-\frac{1}{\sqrt{11}}$ or exact equivalent only. Condone $x\geqslant\frac{1}{\sqrt{11}}$ $x\leqslant-\frac{1}{\sqrt{11}}$, with $x\geqslant1, x\leqslant-1$

Accept exact equivalents such as $x\geqslant\frac{\sqrt{11}}{11}$ $x\leqslant-\frac{\sqrt{11}}{11}$; $|x|\geqslant\frac{1}{\sqrt{11}}$; $\left\{\left(-\infty,-\frac{\sqrt{11}}{11}\right]\cup\left[\frac{\sqrt{11}}{11},\infty\right)\right\}$

Condone the word "and" appearing between the two sets of values.

Withhold the final mark if $x\geqslant\frac{1}{\sqrt{11}}$ $x\leqslant-\frac{1}{\sqrt{11}}$, appears with values not in this region eg $x\leqslant 1, x\geqslant-1$

(ii)

B1 Differentiates and achieves a correct line involving $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$

Accept $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$, $\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{1}{\cos 2y}\times -2\sin 2y$ $\quad 2\sec 2y\tan 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$

M1 For inverting their expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ to achieve an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

The variables (on the rhs) must be consistent, you may condone slips on the coefficients but not the terms.
In the alternative method it is for correctly changing the subject

M1 Scored for using $\tan^2 2y=\pm1\pm\sec^2 2y$ **and** $\sec 2y=\mathrm{e}^x$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

Alternatively they could use $\sin^2 2y+\cos^2 2y=1$ with $\cos 2y=\mathrm{e}^{-x}$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

For the M mark you may condone $\sec^2 2y=\left(\mathrm{e}^x\right)^2$ appearing as e^{x^2}

A1 cso $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$ Final answer, do not allow if students then simplify this to eg. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\mathrm{e}^x-1}$

Condone $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$ but do not allow $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$

Allow a misread on $x=\ln(\sec y)$ for the two method marks only

Question Number	Scheme	Marks
8 (a)	$P_0 = \frac{100}{1+3} + 40 = 65$	B1
		(1)
	$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$	M1
(b)	$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\left(1+3\mathrm{e}^{-0.9t}\right)\times -10\mathrm{e}^{-0.1t} - 100\mathrm{e}^{-0.1t} \times -2.7\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$	M1 A1
		(3)
(c)(i)	At maximum $-10\mathrm{e}^{-0.1t} - 30\mathrm{e}^{-0.1t} \times \mathrm{e}^{-0.9t} + 270\mathrm{e}^{-0.1t} \times \mathrm{e}^{-0.9t} = 0$	
	$\mathrm{e}^{-0.1t}\left(-10 + 240\mathrm{e}^{-0.9t}\right) = 0$	
	$\mathrm{e}^{-0.9t} = \frac{10}{240}$ oe $\mathrm{e}^{0.9t} = 24$	M1
	$-0.9t = \ln\left(\frac{1}{24}\right) \Rightarrow t = \frac{10}{9}\ln(24) = 3.53$	M1, A1
(c) (ii)	Sub $t = 3.53 \Rightarrow P_T = 102$	A1
		(4)
(d)	40	B1
		(1)
		9 marks

(a)

B1 $\left(P_0 =\right)65$

(b)

M1 For sight of $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$ (Allow C =1)This may be within an incorrect product or quotient rule

M1 Scored for a full application of the quotient rule. If the formula is quoted it should be correct.

The denominator should be present even when the correct formula has been quoted.

In cases where a formula has not been quoted it is very difficult to judge that a correct formula has been used (due to the signs between the terms). So…………….

if the formula has not been quoted look for the **order** of the terms $\frac{(1+3\mathrm{e}^{-0.9t})\times p\mathrm{e}^{-0.1t} - q\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t}}{(1+3\mathrm{e}^{-0.9t})^2}$

$\frac{(1+3\mathrm{e}^{-0.9t})\times p\mathrm{e}^{-0.1t} + q\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t}}{(1+3\mathrm{e}^{-0.9t})^2}$

For the product rule. Look for $a\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-1} \pm b\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-2}$ either way around

Penalise if an incorrect formula is quoted . Condone missing brackets in both cases.

A1 A correct **unsimplified** answer.

Eg using quotient rule $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)=\frac{-10\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)+270\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$ oe $\frac{-10\mathrm{e}^{-0.1t}+240\mathrm{e}^{-1t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$ simplified

Eg using product rule $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)=-10\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-1}+270\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-2}$ oe

Remember to isw after a correct (unsimplified) answer.

There is no need to have the $\frac{\mathrm{d}P}{\mathrm{d}t}$ and it could be called $\frac{\mathrm{d}y}{\mathrm{d}x}$

(c)(i) Do NOT allow any marks in here without sight/implication of $\frac{\mathrm{d}P}{\mathrm{d}t}=0, \frac{\mathrm{d}P}{\mathrm{d}t}<0$ OR $\frac{\mathrm{d}P}{\mathrm{d}t}>0$

The question requires the candidate to find t using part (b) so it is possible to do this part using inequalities using the same criteria as we apply for the equality. All marks in (c) can be scored from an incorrect denominator (most likely v), no denominator, or using a numerator the wrong way around ie $uv'-u'v$

M1 Sets their $\frac{\mathrm{d}P}{\mathrm{d}t}=0$ or the numerator of their $\frac{\mathrm{d}P}{\mathrm{d}t}=0$, factorises out or cancels a term in $\mathrm{e}^{-0.1t}$ to reach a form $A\mathrm{e}^{\pm 0.9t}=B$ oe. Alternatively they could combine terms to reach $A\mathrm{e}^{-t}=B\mathrm{e}^{-0.1t}$ or equivalent

Condone a double error on $\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t}=\mathrm{e}^{-0.1t\times -0.9t}$ or similar before factorising. **Look for correct indices**.

If they use the product rule then expect to see their $\frac{\mathrm{d}P}{\mathrm{d}t}=0$ followed by multiplication of $\left(1+3\mathrm{e}^{-0.9t}\right)^2$ before similar work to the quotient rule leads to a form $A\mathrm{e}^{\pm 0.9t}=B$

M1 Having set the numerator of their $\frac{\mathrm{d}P}{\mathrm{d}t}=0$ and obtained either $\mathrm{e}^{\pm kt}=C$ (k may be incorrect) or $A\mathrm{e}^{-t}=B\mathrm{e}^{-0.1t}$ it is awarded for the correct order of operations, taking ln's leading to $t=..$

It cannot be awarded from impossible equations Eg $\mathrm{e}^{\pm 0.9t}=-0.3$

A1 cso $t=$ awrt 3.53 Accept $t=\frac{10}{9}\ln(24)$ or exact equivalent.

(c)(ii)

A1 awrt 102 following 3.53 The M's must have been awarded. This is not a B mark.

(d)

B1 Sight of 40

Condone statements such as $P\to 40$ $k\ldots 40$ or likewise

Question Number	Scheme	Marks
9(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$	
	$= \tan x\cos 2x$	dM1 A1*
		(4)
(b)	$\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$	
	$\cos 2x - 3\sin x = 0$	M1
	$\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$	M1
	$\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3 \pm \sqrt{17}}{4} \Rightarrow x = ...$	M1
	Two of $x = 16.3°, 163.7°, 0, 180°$	A1
	All four of $x = 16.3°, 163.7°, 0, 180°$	A1
		(5)
		(9 marks)

(a)

M1 Uses a correct double angle identity involving $\sin 2x$ Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\sin 2x = 2\sin x\cos x$ and attempts to combine the two terms using a common denominator. This can be awarded on two separate terms with a common denominator.
Alternatively uses $\sin x = \tan x\cos x$ and attempts to combine two terms using factorisation of tan x

dM1 Both M's must have been scored. Uses a correct double angle identity involving $\cos 2x$.

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent
Withhold this mark if for instance they write $\tan x = \dfrac{\sin}{\cos}$

If the candidate $\times \cos x$ on line 1 and/or $\div \sin x$ they cannot score any more than one mark unless they are working with both sides of the equation or it is fully explained.

(b)

M1 The $\tan x$ must be cancelled or factorised out to produce $\cos 2x - 3\sin x = 0$ or $\dfrac{\cos 2x}{\sin x} = 3$ oe Condone slips

M1 Uses $\cos 2x = 1 - 2\sin^2 x$ to form a 3TQ=0 in $\sin x$ The = 0 may be implied by later work

M1 Uses the formula/completion of square or GC with invsin to produce at least one value for x
It may be implied by one correct value.
This mark **can** be scored from factorisation of their 3TQ in $\sin x$ **but only if** their quadratic factorises.

A1 Two of $x = 0, 180°, \text{awrt } 16.3°, \text{awrt } 163.7°$ or in radians two of awrt 0.28, 2.86, 0 and π or 3.14
This mark can be awarded as a SC for those students who just produce $0, 180°$ (or 0 and π) from $\tan x = 0$ or $\sin x = 0$.

A1 All four values in degrees $x = 0, 180°, \text{awrt } 16.3°, \text{awrt } 163.7°$ and no extra's inside the range $0 ,, \; x < 360°$.
Condone $0 = 0.0$ and $180° = 180.0°$ Ignore any roots outside range.

Alternatives to parts (a) and (b)

(a) Alt 1	$\tan x\cos 2x = \tan x\left(2\cos^2 x - 1\right)$	M1
	$= 2\tan x\cos^2 x - \tan x$	
	$= 2\dfrac{\sin x}{\cos x}\cos^2 x - \tan x$	M1
	$= 2\sin x\cos x - \tan x$	
	$= \sin 2x - \tan x$	dM1 A1
		(4)

a) Alt 1 Starting from the rhs

M1 Uses a correct double angle identity for $\cos 2x$. Accept any correct version including $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and attempts to multiply out the bracket

dM1 Both M's must have been scored. It is for using $2\sin x\cos x = \sin 2x$

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent. See Main scheme for how to deal with candidates who $\div\tan x$

(a) Alt 2	$\sin 2x - \tan x \equiv \tan x\cos 2x$	
	$2\sin x\cos x - \tan x \equiv \tan x(2\cos^2 x - 1)$	M1
	$2\sin x\cos x - \cancel{\tan x} \equiv 2\tan x\cos^2 x - \cancel{\tan x}$	
	$2\sin x\cos x \equiv 2\dfrac{\sin x}{\cos x}\cos^2 x$	M1
	$2\sin x\cos x \equiv 2\sin x\cos x$	dM1
	+statement that it must be true	A1*

a) Alt 2 Candidates who use both sides

M1 Uses a correct double angle identity involving $\sin 2x$ **or** $\cos 2x$. Can be scored from either side
Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$ or $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 **Uses** $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and cancels the $\tan x$ term from both sides

dM1 Uses a correct double angle identity involving $\sin 2x$ Both previous M's must have been scored

A1* A fully correct solution with no errors or omissions AND statement "hence true", "a tick", "QED". W[5]
All notation must be correct and variables must be consistent

....

It is possible to solve part (b) without using the given identity. There are various ways of doing this, one of which is shown below.

$$\sin 2x - \tan x = 3\tan x\sin x \Rightarrow \quad 2\sin x\cos x - \frac{\sin x}{\cos x} = 3\frac{\sin x}{\cos x}\sin x$$

$2\sin x\cos^2 x - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ and $\cos x$

$2\sin x\left(1-\sin^2 x\right) - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ only

$\left(2\sin^2 x + 3\sin x - 1\right)\sin x = 0$

$x = ..$ — M1 Solving equation to find at least one x

Two of $x = 16.3°, 163.7°, 0, 180°$ — A1

All four of $x = 16.3°, 163.7°, 0, 180°$ and no extras A1

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